

4.2/32

$$e^{2x} - e^x = 6$$

$$(e^x)^2 - e^x = 6$$

$$u^2 - u = 6$$

$$u^2 - u - 6 = 0$$

$$(u-3)(u+2) = 0$$

$$u = -2, +3$$

$$\left\{ \begin{array}{l} \text{let} \\ u = e^x \end{array} \right.$$

$$e^x = -2, +3$$

$$\left\{ \begin{array}{l} e^x = 3 \\ \ln(e^x) = \ln 3 \end{array} \right.$$

$$e^x = -2?$$

$$e^x = +3$$

~~⊗~~

$$\begin{array}{l} y_1 = e^x \\ y_2 = 3 \end{array}$$

$$x = 1.0986 \dots$$

$$= \ln 3$$

graph
 3^x

graph
 2^x

$$m_{\text{tan}} = \text{at } x=0$$

2ND
DRAW
tangent
 $x=0$

$$m_{\text{tan}} = \text{at } x=0$$

1.0986
graph

graph
 5^x

base	m
1.5	.4054
2	.6931
3	1.0986
5	1.6094

.6931

graph
 $(1.5)^x$

$$m_{\text{tan}} = 1.6094 \text{ at } x=0$$

$$m_{\text{tan}} = .4054 \text{ at } x=0$$

Welcome to "e"

2.718281828459045...

graph
 e^x

$$e^{x+y} = e^x \cdot e^y$$

$$\ln(xy) = \ln x + \ln y$$

$$e^{x-y} = \frac{e^x}{e^y}$$

$$\ln \frac{x}{y} = \ln x - \ln y$$

$$(e^x)^y = e^{yx}$$

$$\ln x^y = y \ln x$$

$$e^x = 3$$

$$\ln(e^x) = \ln 3$$

$$x(\ln e) = \ln 3$$

$$x = \ln 3$$

$$\ln_b b = 1$$

4.2 / 9 a

$$\ln(a^2 \cdot \sqrt{bc}) =$$

$$\ln(a^2) + \ln \sqrt{bc}$$

$$2 \ln a + \ln(bc)^{1/2}$$

$$2 \ln a + \frac{1}{2} \ln(bc)$$

$$2 \ln a + \frac{1}{2} (\ln b + \ln c)$$

$$2r + \frac{s}{2} + \frac{t}{2}$$

logarithms
turn
times
into
plus

4.2/ab

$$\ln\left(\frac{b}{a^3c}\right)$$

$$\ln b - \ln(a^3c)$$

$$= \ln b - (3\ln a + \ln c)$$

$$= s - (3r + t) = s - 3r - t$$

4.2/19

$$\ln\left(\frac{1}{x}\right) = -2$$

$$\frac{1}{e^2} = e^{-2} = \frac{1}{x}$$

$$x = e^2$$

using alternative
exponential
form

$$\begin{aligned} a^b &= c \\ \Leftrightarrow \\ \ln_b c &= a \end{aligned}$$

$$e^{\left(\ln\left(\frac{1}{x}\right)\right)} = e^{-2}$$

$$\begin{aligned} e^{\ln a} &= a \\ \ln e^b &= b \end{aligned}$$

$$\frac{1}{x} = \frac{1}{e^2} \Rightarrow x = e^2$$

$$\underline{4.2/29)} \quad 2e^{3x} = 7$$

$$\ln(2e^{3x}) = \ln(7)$$

$$\ln 2 + 3x(\underline{\ln e}) = \ln 7$$

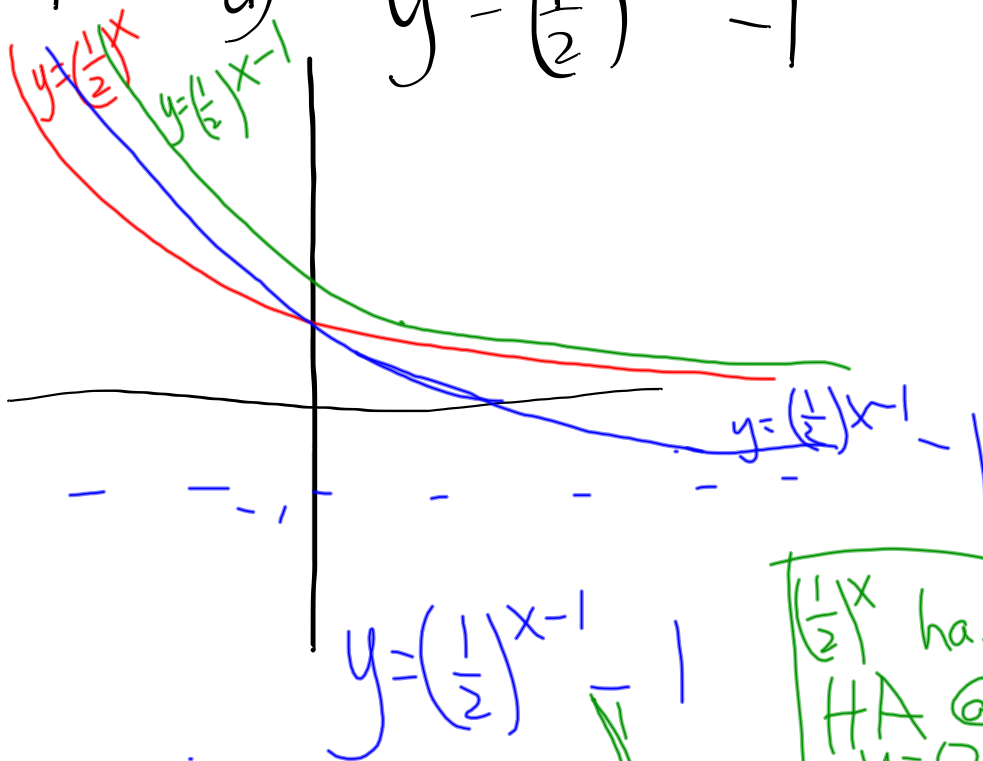
$$3x = \ln 7 - \ln 2$$

$$x = \frac{1}{3}(\ln 7 - \ln 2) = \frac{1}{3}\ln\left(\frac{7}{2}\right)$$

4.2/35

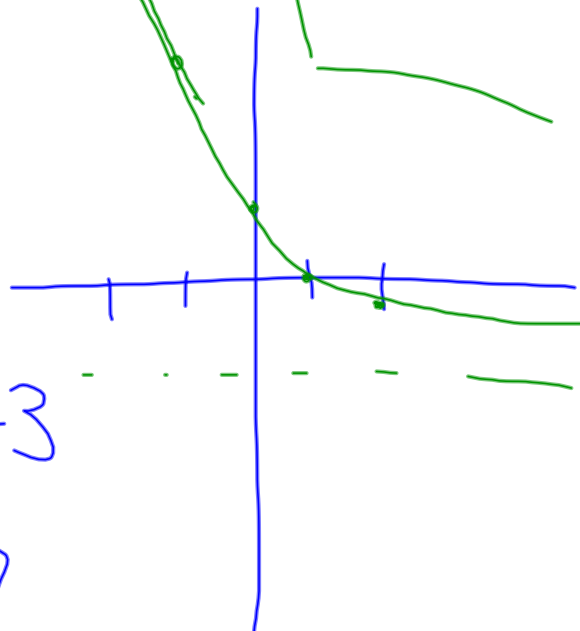
a)

$$y = \left(\frac{1}{2}\right)^{x-1} - 1$$



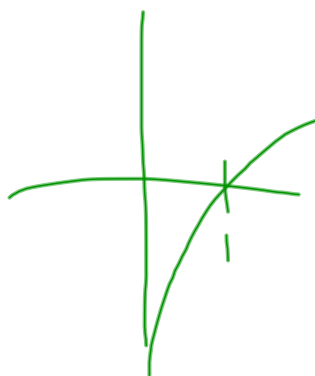
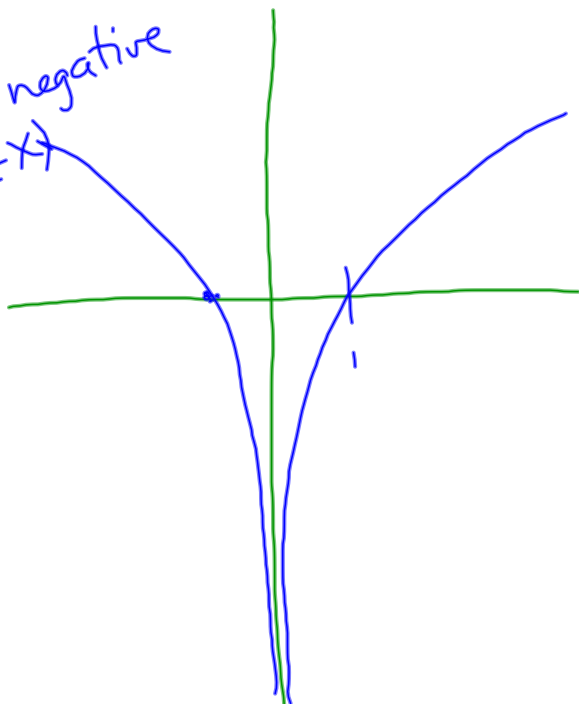
$\left(\frac{1}{2}\right)^x$ has a
HA @
 $y = 0$

x	$\left(\frac{1}{2}\right)^{x-1} - 1$
2	$-\frac{1}{2}$
1	0
0	1
-1	$\left(\frac{1}{2}\right)^{-2} - 1 = 2^2 - 1 = 3$
-2	$\left(\frac{1}{2}\right)^{-3} - 1 = 8 - 1 = 7$



4.2/35b $y = \ln|x|$

x is negative
 $\ln(-x)$



x is positive
graph $\ln x$

$$\frac{d}{dx}(e^x) = e^x \quad \star$$

$$\frac{d}{dx}(b^x) = \frac{d}{dx}((e^{\ln b})^x) =$$

$$\frac{d}{dx}(1x) = 1 \quad = \frac{d}{dx}(e^{x \ln b})$$

$$\frac{d}{dx}(2x) = 2 \quad = (e^{x \ln b}) \left(\frac{d}{dx}(x \ln b) \right)$$

$$\frac{d}{dx}(3x) = 3$$

$$= b^x \cdot \ln b$$

$$\frac{d}{dx}((\ln b)x) = \ln b$$



$$(e^{\ln x}) = x$$

$$(e^{\ln x})(\ln x)' = 1 \Rightarrow$$

$$x (\ln x)' = 1 \Rightarrow (\ln x)' = \frac{1}{x}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$$

$$\log_b x = \frac{\ln x}{\ln b}$$

WARNING

$$\frac{d}{dx}(x^b) = bx^{b-1}$$

$$\frac{d}{dx}(b^x) = b^x (\ln b)$$

$$4.3/3 \quad y = (\ln x)^2$$

$$y' = 2(\ln x)' \left(\frac{1}{x} \right) \\ = \frac{2 \ln x}{x}$$

$$y = (\ln x)(\ln x)$$

$$y' = \left(\frac{1}{x} \right)(\ln x) + (\ln x) \left(\frac{1}{x} \right) \\ = 2(\ln x) \left(\frac{1}{x} \right) = \frac{2 \ln x}{x}$$

$$4.2/32) \quad \left. \begin{aligned} e^{2x} - e^x &= 6 \\ (e^x)^2 - e^x &= 6 \\ u^2 - u &= 6 \\ u^2 - u - 6 &= 0 \end{aligned} \right\} \begin{aligned} \text{let} \\ u = e^x \end{aligned}$$

$$(u-3)(u+2) = 0$$

$$e^x = u = -2, +3$$

$$e^x = -2, 3$$

$$\begin{array}{l} e^x = -2 \\ e^x = 3 \end{array} \quad y_1 = e^x$$

$$\cancel{\emptyset} \quad \ln(e^x) = \ln(3) \quad y_2 = 3$$

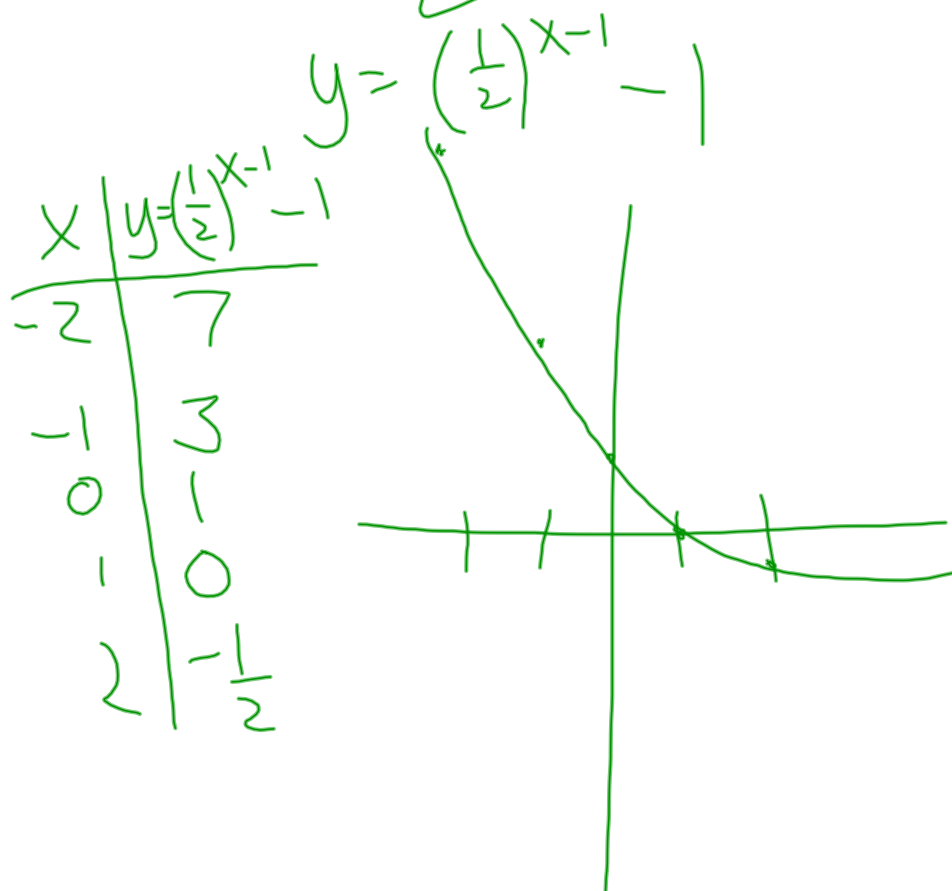
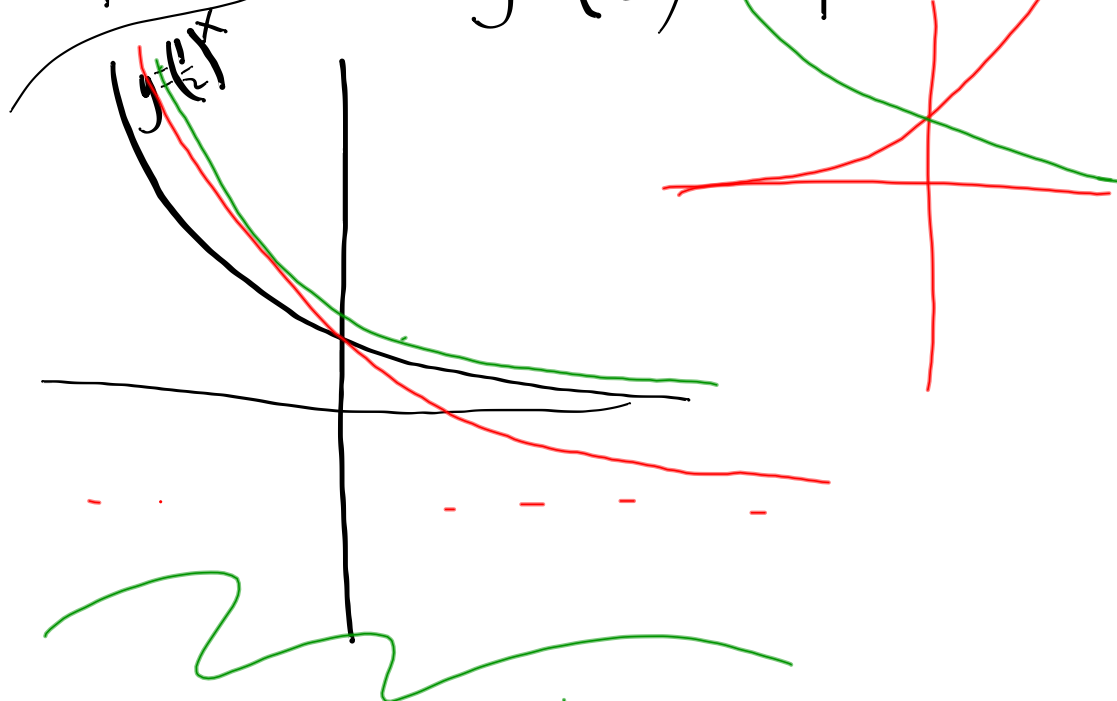
$$\boxed{x = \ln 3} \quad x = 1.0986 \dots$$

$$\ln 3 = \dots$$

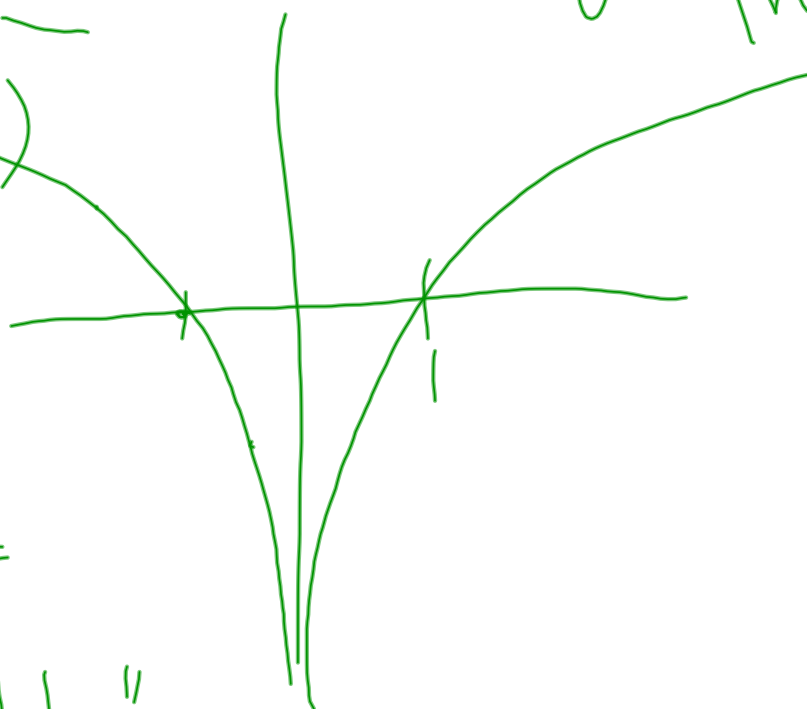
$$x(\ln e)$$

$$\log_e e = 1$$

4.2/35) a) $y = \left(\frac{1}{2}\right)^{x-1} - 1$



35b $y = \ln |x|$ when $x > 0$
 $\ln(-x)$ when $x < 0$



$\ln b =$
 $\log_e b =$
 $= \text{"natural log"}$

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4.2

$$\log_{10} \sqrt{x} = -1$$

$$10^{-1} = \sqrt{x}$$

$$\frac{1}{10} = \sqrt{x}$$

$$\frac{1}{100} = x$$

$$\log_{10} x^{1/2} =$$

$$\frac{1}{2} \log_{10} x = -1$$

$$\log_{10} x = -2$$

$$10^{-2} = x$$

$$e^{x+y} = e^x \cdot e^y$$

$$\log_b x \cdot y = \log_b x + \log_b y$$

$$e^{x-y} = \frac{e^x}{e^y}$$

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

$$(e^x)^y = e^{yx}$$

$$\log x^y = y \log x$$

logarithms change multiplication
to addition

and divides
to subtraction

New Stuff.

$$\underbrace{\frac{d}{dx}(e^x) = e^x} \quad \left\{ \begin{array}{l} \frac{d}{dx^7}(e^x) = e^x \end{array} \right.$$

$$\frac{d}{dx}(b^x) = \frac{d}{dx}(e^{x \ln b})$$

$$\log_b b^n = n$$

$$b^{(\log_b n)} = n$$

$$\frac{d}{dx}(1 \cdot x) = 1 \quad \text{equals} \quad \frac{d}{dx}(e^{x \ln b}) =$$

$$\frac{d}{dx}(2 \cdot x) = 2 \quad e^{x \ln b} \cdot \left(\frac{d}{dx} x \ln b \right)$$

$$\frac{d}{dx}(3 \cdot x) = 3 \quad = (e^{x \ln b})(\ln b)$$

$$\frac{d}{dx}(b^x) = \underline{(b^x)}(\ln b)$$

e^x and $\ln x$ are inverses.

$$\underbrace{e^{\ln x} = \ln(e^x) = x}$$

$$e^{\ln x} = x$$

$$(e^{\ln x})(\ln x)' = 1$$

$$\frac{d}{dx}(\ln x) = \frac{1}{e^{\ln x}} = \frac{1}{x}$$

$$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$$

$$\log_b x = \frac{\ln x}{\ln b} \dots \log_b x = y \quad b^y = x$$

"chng of base formula"

$$\ln(b^y) = \ln x$$

$$y \ln b = \ln x$$

$$4.3/1 \quad \frac{d}{dx}(\ln(2x))$$

$$\ln(2x)$$

$$= \ln 2 + \ln x$$

$$\frac{1}{(2x)}(2) = \frac{1}{x}$$

4.3/3)

$$\frac{d}{dx}(\ln x)^2$$

$$\frac{d}{dx}(\ln x)(\ln x)$$

$$= \left(\frac{1}{x}\right)(\ln x) + (\ln x)\left(\frac{1}{x}\right)$$

$$= 2(\ln x)\left(\frac{1}{x}\right)$$

$$= 2\left(\frac{1}{x}\ln x\right)$$

$$= \frac{2 \ln x}{x}$$

WARNING

$$\frac{d}{dx}(x^b) = bx^{b-1}$$

$$\frac{d}{dx}(b^x) = b^x (\ln b)$$